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Ranked Set Sampling Method: An Overview

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Abstract

Ranked set sampling (RSS) is a statistical technique in which multiple random samples are drawn, ranked with respect to characteristic of interest and then specific units from each sample are quantified to estimate the population mean. This method is a cost-effective sampling procedure designed to provide more efficient estimators of population parameters such as mean and can be more powerful than Simple Random Sampling (SRS) even in the presence of ranking errors. It was originally introduced by McIntyre (1952), this method is especially useful when measuring sampling units is difficult or expensive, but visual ranking them or expert judgment or any cheap auxiliary variable is easy. In this method, after selecting m^2 units randomly from an infinite population and arranged them into m sets each containing m units, a unit is randomly selected from each set. Each so obtained unit is then ranked between 1 and m (both inclusive) with respect to variable of interest or on the basis of highly correlated auxiliary variable is then quantified. Sample is selected through a pre-assigned pattern as from the first set, select the smallest unit; from the second set, select the second smallest unit and this process is continued until the largest unit is selected from the m^{th} set. For obtaining larger set size repeat the entire process r times to obtain a sample size of $n = mr$. Performance of RSS further improves when appropriate unequal allocation is used instead of equal allocation based on Neyman's approach with the same set size. The ranked set sample mean is an unbiased estimator of the population mean even with a smaller variance than the simple random sample mean of the same size. The **c**ost-effectiveness of RSS achieved because only a smaller portion of units mr are measured, so costs are lower than taking a full m^2r sample are used as in the SRS. The present paper focuses on the overview and continued development on the RSS estimator and its broad applications particularly effective in agriculture, forestry, ecological studies and in environmental studies where visual ranking is possible. These procedures are illustrated using a real data set regarding the yield of potato. The technique is more useful to those who look for cost-effective technique for estimating agricultural products that are grown underground such as potato, ginger, turmeric, garlic, onion, beetroot, peanut, etc.

***Keywords*:** Equal allocation, unequal allocation, cost-effective, relative precision, relative cost.

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1. Introduction

The Ranked set sampling (RSS) was first proposed by McIntyre (1952) to increase the efficiency of the estimation of population mean. The method is useful when the variable of interest is expensive or difficult to measure but it can be ranked easily at a negligible cost without using their actual measurements. It is a cost effective sampling procedure as compared with the commonly used Simple Random Sampling (SRS). He suggested the sample mean based on RSS as an estimator of the population mean and shown that estimator based on RSS is more efficient than SRS. There have been many new modifications and improvements have been made on RSS since its inception by McIntyre. RSS performs better when perfect ranking scenario is done, but if we use imperfect or concomitant ranking, then in these situations also RSS performs better than SRS. An error get involved during ranking due to dependence on the ranker's judgment, Dell and Clutter (1972) showed that the RSS estimator of a population mean remains unbiased and is at least as efficient as the SRS estimator with the same number of quantifications. Kaur *et al.* (1997) showed that RSS performs better than SRS when units corresponding to each rank are allocated equally. The performance of RSS further improves when unequal allocation is used instead of equal allocation; see Patil, Sinha and Taillie (1994b). Takahasi and Wakimoto (1968) shown that the relative precision (RP) of RSS compared with SRS lies between 1 and $(m+1)/2$ for equal allocation whereas they have shown that the RP of RSS relative to SRS lies between 0 and m in case of unequal allocation, where m is the set size. In the case of RSS with an unequal allocation, McIntyre (1952) proposed the application of optimum allocation based on Neyman's approach where the sampling units are allocated to various ranks order in proportional to the standard deviation of that rank order. When a concomitant variable is available, ratio estimation based on ranked set sampling was proposed. This ratio estimator is more efficient than SRS; see Al-Hadhrami (2009). For skewed distributions, Takahasi (1970) developed an RSS method that uses the ranking information at the estimation stage unlike McIntyre's RSS that needs the information at the selection stage.

The present paper describes an overview on RSS estimator and effectiveness of RSS in case of equal as well unequal allocation as compared with SRS estimator. Here, section 2 describes the RSS with equal allocation and section 3 explains the RSS with an unequal allocation for skewed population. Section 4 shows the use of Takahasi's RSS method and modified Takahasi's RSS method. Section 5 illustrates the RSS method from the use of RSS with equal and unequal allocation for estimating the tuber weight based on real data set taken from The Central Potato Research Station (Indian Council of Agricultural Research), Patna, Bihar, India. In section 6, the results and discussion of the present work are included.

2. RSS with Equal Allocation

In RSS procedure with equal allocation, m^2 units are randomly selected from an infinite population and then the selected units are arranged in m sets each consisting of m units. After ranking the units of each set separately with respect to the variable of interest, the unit with the lowest rank is selected from the first set, the unit with the second lowest rank is quantified from the second set and the process is continued until the unit possessing rank m is quantified from the m^{th} set. In this way m units are quantified out of the initially m^2 units selected from an infinite population. The whole process is repeated r times in order to get a large sample size n. This provides a ranked set sample of size $n = mr$. We call m as the set size and r as the number of cycles. As one gets the same number of quantifications r for each rank order this procedure is called RSS with the equal allocation (RSSWEA) [Norris, Patil and Sinha, 1995].

In general, suppose that $X_{(i:m)j}$ denotes the i^{th} order statistic based on perfect ranking in the j^{th} cycle, for $i = 1,2,...,m$ and $j = 1,2,...,r$. It is to be noted that these are not *iid* in general, but for a given value of i these are so with $E(X_{(i:m)j}) = \mu_{(i:m)}$ and $Var(X_{(i:m)j}) = \sigma^2_{(i:m)}$.

The McIntyre's estimator $\hat{\mu}_{MRSS}$ of the population mean μ is defined as follows:

$$\hat{\mu}_{MRSS} = \frac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)} \qquad \text{where, } \hat{\mu}_{(i:m)} = \frac{1}{r}\sum_{j=1}^{r} X_{(i:m)j}$$

Here, $E(\hat{\mu}_{(i:m)}) = \mu_{(i:m)}$; $E(\hat{\mu}_{MRSS}) = \mu$ and $Var(\hat{\mu}_{(i:m)}) = \dfrac{\sigma^2_{(i:m)}}{r}$.

Thus we get, $Var(\hat{\mu}_{MRSS}) = Var\left[\dfrac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)}\right] = \dfrac{1}{m^2}\sum_{i=1}^{m}Var(\hat{\mu}_{(i:m)}) = \dfrac{1}{m^2}\sum_{i=1}^{m}\dfrac{\sigma^2_{(i:m)}}{r}$

Therefore, $Var(\hat{\mu}_{MRSS}) = \dfrac{1}{m^2 r}\sum_{i=1}^{m}\sigma^2_{(i:m)}$.

The estimate of σ^2 for simple random sampling is given by,

$$\hat{\sigma}^2_{MRSS} = \left[\frac{mr - m + 1}{m^2 r(r-1)}\right]\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i:m)j} - \hat{\mu}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\hat{\mu}_{(i:m)} - \mu_{MRSS})^2$$

Relative Precision (RP) for an Equal Allocation

The relative precision (RP) of the MRSS estimator $\hat{\mu}_{MRSS}$ as compared with SRS estimator $\hat{\mu}_{SRS}$ with the same sample size n is obtained as:

$$RP = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSS})}$$

As $\quad \mathrm{var}(\hat{\mu}_{SRS}) = \frac{\sigma^2}{mr}$, this leads to

$$RP = \frac{\sigma^2}{\overline{\sigma^2}} \qquad \text{where } \overline{\sigma^2} = \frac{\sum_{i=1}^{m} \sigma^2_{(i:m)}}{m}$$

An equivalent and useful measure could be relative cost (RC) and relative savings (RS). These are defined as $RC = \frac{1}{RP}$ and $RS = 1 - RC$

The range of RP is $1 \le RP \le \frac{m+1}{2}$ and that of RS is $0 \le RS \le \frac{m-1}{m+1}$.

3. RSS with Unequal Allocation

For unequal allocation McIntyre (1952) suggested that to take sample size of each rank order proportional to its standard deviation while sampling with asymmetrical populations. It implies that if r_i denotes the number of the sets having quantified units with rank i, then $r_i \alpha \sigma_{(i:m)}$ for $i = 1,2,\ldots,m$. Thus the expression for r_i is given as

$$r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m} \sigma_{(i:m)}}; \quad i = 1,2,\ldots,m.$$

The RSS estimator $\hat{\mu}_{MRSSUA}$ of the population mean μ based on an unequal allocation of samples is given by

$$\hat{\mu}_{MRSSUA} = \frac{1}{m}\sum_{i=1}^{m} \frac{T_i}{r_i} \qquad \text{and} \qquad Var(\hat{\mu}_{MRSSUA}) = \frac{1}{m^2}\sum_{i=1}^{m} \frac{\sigma^2_{(i:m)}}{r_i}$$

where T_i represents the sum of the quantifications of the r_i units having i^{th} rank order. On putting the value of r_i into the expression for $Var(\hat{\mu}_{MRSSUA})$ we have

$$Var(\hat{\mu}_{MRSSUA}) = \frac{(\bar{\sigma})^2}{n}; \qquad \text{where, } \bar{\sigma} = \frac{\sum_{i=1}^{m} \sigma_{(i:m)}}{m}$$

The estimate of σ^2 for simple random sampling in case of unequal allocation is given by,

$$\hat{\sigma}^2_{MRSSUA} = \sum_{i=1}^{m}\left[\frac{m(r_i-1)+1}{m^2 r_i(r_i-1)}\right]\sum_{j=1}^{r_i}(X_{(i:m)j}-\overline{X}_{(i:m)})^2+\left(\frac{1}{m}\right)\sum_{i=1}^{m}(\overline{X}_{(i:m)}-\overline{X}_{(m)r})^2$$

Relative precision (RP) for unequal allocation

The relative precision (RP_{ua}) of $\hat{\mu}_{MRSSUA}$ relative to $\hat{\mu}_{SRS}$ with the same number of quantification is given by

$$RP_{ua} = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSSUA})}$$

This yields that
$$RP_{ua} = \frac{\sigma^2/n}{\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r_i}/m}$$

This could also be written as
$$RP_{ua} = \left(\frac{\sigma}{\overline{\overline{\sigma}}}\right)^2.$$

This proves that $RP_{ua} \geq RP$. Takahasi and Wakimoto (1968) show that $0 \leq RP_{ua} \leq m$. For the above results we can see Sinha (2005).

4. Takahasi's RSS Estimators and Modified Takahasi's RSS Estimators

Takahasi's RSS Estimators:

For this estimator we need to obtain a sample of size (*n* - *m*) by Takahasi's method and one cycle of data through McIntyre's method. The estimator of the population mean μ is given by

$$\hat{\mu}_{TRSS} = \frac{1}{m}\sum_{i=1}^{m}\frac{S_i}{r_i}$$

where S_i denotes the sum of the quantifications having rank *i* and $r_i \geq 1$ shows the frequency of the i^{th} ordered quantification.

The variance of the $\hat{\mu}_{TRSS}$ is given by

$$Var(\hat{\mu}_{TRSS}) = \frac{1}{m(mr-m+1)}\left[1-\left(1-\frac{1}{m}\right)^{mr-m+1}\right]\sum_{i=1}^{m}\sigma^2_{(i:m)}$$

Modified Takahasi's RSS Estimators

For this estimator we draw an RSS of size (*n* - *2m*) based on Takahasi's method and two cycles of data are obtained following McIntyre's method. The estimator of the population mean μ when $r_i \geq 2$ is given by

$$\hat{\mu}_{MTRSS} = \frac{1}{m}\sum_{i=1}^{m}\frac{S_i}{r_i}$$

The variance of the estimator $\hat{\mu}_{MTRSS}$ is given by

$$Var(\hat{\mu}_{MTRSS}) = \frac{1}{m(mr-2m+1)}\left[1-\frac{m}{mr-2m+2}\left\{1-\left(1-\frac{1}{m}\right)^{mr-2m+2}\right\}\right]\sum_{i=1}^{m}\sigma^2_{(i:m)}$$

5. Illustration

For an illustration, a real data set of four characteristics of potato plant fresh haulm weight (in kg), fresh root weight (in kg), tuber (potato) weight (in kg) and chlorophyll measurements of the plant each having 96 observations are taken, see (Kumar and Sinha, 2015, 2020),. For estimating tuber weight, we have used perfect and concomitant ranking scenario both. For this, the plots have been ranked using the exact weight of potato which is supposed to be known. We have randomly selected sixteen plots from the given data set and weights of the tuber are presented in four rows and four columns. The procedure is carried out for all the three sets. For using RSS with unequal allocation, we compute r_i using the formula $r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m}\sigma_{(i:m)}}$ where $i = 1,2,...,m$. The results obtained are given below. Table 1 represents the weights of tuber in four plots of equal size in each row for three sets.

Table 1: RP, RC and RS of tuber under Perfect Ranking Scenario

Allocation and Method	**RP**	**RC**	**RS**
Perfect Ranking (Equal Allocation) ($m = 4, r = 3, n = 12$)	1.70	0.59	0.41 (41%)
Perfect Ranking (Unequal Allocation), ($m = 4, n = 12$)	2.21	0.45	0.55 (55%)

Concomitant Ranking Scenario of tuber weight Based on Chlorophyll Measure

The concomitant ranking is performed using the ranks on the basis of chlorophyll level, fresh haulm weight and on the basis of two characteristics chlorophyll level and fresh haulm weight jointly. The main characteristic of interest is tuber weight while other characteristics are used as concomitant variables for ranking the plots with respect to the main characteristic of interest. But for selecting concomitant variable for ranking we need to choose those variables which are highly correlated with the tuber weight. For this we performed the correlation analysis using Statistical Software, Minitab 16. The output of Correlation Analysis is given below.

Correlation Matrix:

Correlations: Fresh Haulm Weight (Kg), Fresh Root Weight (gm), Tubers Weight (Kg), Chlorophyll Measure

```
                        Fresh Haulm Weight(kg) Fresh Root Weight(gm) Tubers Weight(kg)

Fresh Root Weight(gm)         0.085
                              0.410

Tubers Weight(kg)             0.219                0.016
                              0.032                0.875

Chlorophyll Measure           0.049                0.007              0.349
                              0.637                0.945              0.000

Cell Contents: Pearson correlation
               P-Value
```

Table 2: RP, RC and RS of tuber under Concomitant Ranking Scenario

Allocation and Method	RP	RC	RS
Concomitant Ranking (Equal Allocation) ($m = 4, r = 3, n = 12$)	1.52	0.65	0.35 (35%)
Concomitant Ranking (Unequal Allocation) ($m = 4, n = 12$)	1.72	0.58	0.42 (42%)

Modified Takahasi's RSS Method:

Table 3: RP, RC and RS under of tuber under modified Takahasi's Method

Allocation and Method	RP	RC	RS
Modified Takahasi's method as compared with *SRS* ($m = 3, n = 27$)	1.45	0.69	0.31 (31%)
Modified Takahasi's method as compared with *MRSS* ($m = 3, n = 27$)	1.98	0.51	0.49 (49%)

6. Results and Discussion

The results obtained for RP, RC and RS with equal and unequal allocation under perfect and concomitant ranking scenarios and under Modified Takahasi's method are enumerated in Table 1, 2 and 3 given above. From Table 1, 2 and 3, we observe that the RP is obtained higher in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario for the same set size $m = 4$ and the same sample size $n = 12$. It indicates that RSS with unequal allocation performs better than SRS and RSS with equal allocation. Thus, it has also been seen that, RC is less in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario and consequently it observed that RS is higher in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario for the same set size $m = 4$ and the same sample size $n = 12$. Again in the case of Modified Takahasi's method, the relative saving (RS) is 31% as compared to SRS under unequal allocation and RS is 49% as compared to MRSS under unequal allocation for the same set size and same sample size. Thus, the Modified Takahasi's method appears to be more efficient than Simple Random Sampling and McIntyre's Ranked Set Sampling with the same set size and same sample size and is useful to those who look for estimating farm produces that are grown under the land surface like potato, ginger, turmeric, garlic, onion, beetroot, peanut, etc.

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